Dinajpur Govt. college, Dinajpur.

Mathematics Tutorial Exam-2008

THEORY OF NUMBERS

1. Decide whether the congruent $x^2 \equiv 105(\bmod 317)$ is solvable or not.
2. If $s > 1$ then $\zeta(s) = \prod_P \frac{1}{1-P^{-s}}$
3. Define Riemann zeta function. Prove that : $\frac{1}{h(s)} = \sum_{n=1}^{\infty} \frac{\mu(n)}{n^s}, s > 1$ [04]
4. Write down a formula for $\pi(x) - \pi(\sqrt{x})$ intenms of the primes upto $\sqrt{x}$, and use it to determine $\pi(100)$.
5. Define $\theta(x)$ and $\psi(x)$ with example. Show that $\psi(x) - \theta(x) < \frac{\sqrt{x}(\log x)^2}{2\log l}$ [06]

Fluid Dynamics

6. Show that in the two dimensional motion of a liquid the stream function Ψ satisfies the equation $\left(\gamma\nabla^2 - \frac{\partial}{\partial t}\right)\nabla^2\Psi = \frac{\partial(\Psi, \nabla^2\Psi)}{\partial(x, y)}$ [04]
7. Show that for parallel flow through a straight channel the solution of Navier-Stoke's equation is given by $u = -\frac{1}{2\mu}\frac{dp}{dx}\left(b^2 - y^2\right)$ [05]
8. Show that for "Hagen-Poiseuille" flow the volume rate is $\frac{\pi a^4(p_1 - p_2)}{8\mu l}$. [05]
9. Show that the solutions for velocity and pressure in the case of very slow motions of a sphere in an incompressible viscous fluid is given by : $u = U_0\left[\frac{3ax^2}{4r^3}\left(\frac{a^2}{r^2} - 1\right) + 1 - \frac{a}{4r}\left(3 + \frac{a^2}{r^2}\right)\right]$
10. Show that for creeping motion, $\nabla^2 P = 0, \nabla^4\Psi = 0$

LATTICE THEORY

11. Prove that every convex sublattice of a lattice L is the intersection of an ideal and a dual ideal.
12. Two lattices L_1 and L_2 respectively are modular iff $L_1 \times L_2$ is modular.
13. State and prove minimax theorem.
14. A metric lattice L is distributive if and only if $\forall x, y, z \in L$;
    $v(x \vee y \vee z) - v(x \wedge y \wedge z) = v(x) + v(y) + v(z) - v(x \wedge y) - v(y \wedge z) - v(z \wedge x)$
15. Define a Boolean algebra. Show that in algebra B, (i) $(x \wedge y)' = x' \vee y'$ (ii) $(x \vee y)' = x' \wedge y'$

DIFFERENTIAL AND INTEGRAL EQUATIONS

16. Discuss the existence and uniqueness of a solution of the IVP $\frac{dy}{dx} = xy^3, y(0) = 1$. Solve the IVP and fine the interval of existence.
17. Define and compute a fundamental matrix for the system $x_1' = x_1 - 2x_2 - x_3$, $x_2' = -x_1 + x_2 + x_3$, $x_3' = x_1 - x_3$.
18. Prove that the zero solution of $x'' + x = 0$ is uniformly stable, but not asymptotically stable.
19. Show that $\phi(x) = xe^x$ is a solution of the VIE $\phi(x) = \sin x + 2\int \cos(x - \xi)\phi(\xi)d\xi$
20. Solve : $x'(t) = \begin{pmatrix} 3 & -1 \\ 4 & -1 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ t \end{pmatrix}, x(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$